# SM3 2.3: Polynomial Long Division Key

*Vocabulary*: Polynomial, Descending Order, Dividend, Divisor, Quotient, Remainder

*Vocabulary Problems*: Answer the questions about each function:

1) $a(x) = \frac{4x^2+2x+3}{x-8}$

The dividend of $a(x)$ is: $4x^2 + 2x + 3$

The divisor of $a(x)$ is: $x - 8$

2)
$$\begin{array}{r} x+4 \\ x-1\,\overline{)\,x^2+3x+7} \\ -(x^2\ -x) \\ \hline 4x+7 \\ -(4x-4) \\ \hline 11 \end{array}$$

The dividend is: $x^2 + 3x + 7$
The divisor is: $x - 1$
The remainder is: $11$
The $\frac{remainder}{divisor}$ is: $\frac{11}{x-1}$
The quotient is: $x + 4 + \frac{11}{x-1}$

*Setting up*: Set up the long divisions with appropriate placeholders, but do not find the quotients:

3) $\frac{75x^4 - 3}{x^2 - 4x - 8}$

$x^2 - 4x - 8\,\overline{)\,75x^4 + 0x^3 + 0x^2 + 0x - 3}$

4) $\frac{2 - 5x + x^3}{1 - 3x^2}$

$-3x^2 + 0x + 1\,\overline{)\,x^3 + 0x^2 - 5x + 2}$

*Problems*: Find each quotient.

5) $\frac{x^2 + 10x + 24}{x + 6}$

$$\begin{array}{r} x+4 \\ x+6\,\overline{)\,x^2+10x+24} \\ -(x^2\ +6x) \\ \hline 4x+24 \\ -(4x+24) \\ \hline 0 \end{array}$$

6) $\frac{x^2 - 25}{x - 5}$

$$\begin{array}{r} x+5 \\ x-5\,\overline{)\,x^2+0x-25} \\ -(x^2-5x) \\ \hline 5x-25 \\ -(5x-25) \\ \hline 0 \end{array}$$

7) $\frac{5x^2 + 8x + 7}{x + 4}$

$$\begin{array}{r} 5x-12+\frac{55}{x+4} \\ x+4\,\overline{)\,5x^2+8x+7} \\ -(5x^2\ +20x) \\ \hline -12x+7 \\ -(-12x-48) \\ \hline 55 \end{array}$$

8) $\dfrac{x^3+4x^2+9x+36}{x+4}$

$$x+4\,\overline{)\,x^3+4x^2+9x+36}$$
$$x^2+9$$
$$-(x^3+4x^2)$$
$$0+9x+36$$
$$-(9x+36)$$
$$0$$

9) $\dfrac{2x^3-2x^2-x-3}{x-1}$

$$x-1\,\overline{)\,2x^3-2x^2-x-3}$$
$$2x^2-1-\frac{4}{x-1}$$
$$-(2x^3-2x^2)$$
$$0-x-3$$
$$-(-x+1)$$
$$-4$$

10) $\dfrac{x^3+5x}{x^2-2}$

$$x^2+0x-2\,\overline{)\,x^3+0x^2+5x+0}$$
$$x+\frac{7x}{x^2-2}$$
$$-(x^3+0x^2-2x)$$
$$7x+0$$

11) $(x^2+5x-7)\div(x-2)$

$$x-2\,\overline{)\,x^2+5x-7}$$
$$x+7+\frac{7}{x-2}$$
$$-(x^2-2x)$$
$$7x-7$$
$$-(7x-14)$$
$$7$$

12) $(x^2+7)(x+3)^{-1}$

$$x+3\,\overline{)\,x^2+0x+7}$$
$$x-3+\frac{16}{x+3}$$
$$-(x^2+3x)$$
$$-3x+7$$
$$-(-3x-9)$$
$$16$$

13) $(x^3-x^2+x-2)(x^2+3x-1)^{-1}$

$$x^2+3x-1\,\overline{)\,x^3-x^2+x-2}$$
$$x-4+\frac{14x-6}{x^2+3x-1}$$
$$-(x^3+3x^2-x)$$
$$-4x^2+2x-2$$
$$-(-4x^2-12x+4)$$
$$14x-6$$

14) $\dfrac{x^4-1}{x-1}$

$$x-1\,\overline{)\,x^4+0x^3+0x^2+0x-1}$$
$$x^3+x^2+x+1$$
$$-(x^4-x^3)$$
$$x^3+0x^2$$
$$-(x^3-x^2)$$
$$x^2+0x$$
$$-(x^2-x)$$
$$x-1$$
$$-(x-1)$$
$$0$$

15) $\dfrac{2x-5}{x^2-3x-7}$

$$x^2-3x-7\,\overline{)\,2x-5}$$
$$0+\frac{2x-5}{x^2-3x-7}$$

The denominator has more power than the numerator. When divided, the numerator is the remainder and the quotient is the original problem!

*Composite Problems*: Find each quotient.

16) $\dfrac{(x^2+4)+(2x^2+x-5)}{x+10}$

$\dfrac{3x^2+x-1}{x+10}$

$$
\begin{array}{r}
3x-29+\dfrac{289}{x+10} \\
x+10\,\overline{)\,3x^2+\ x-1} \\
-(3x^2+30x) \\
\hline
-29x-1 \\
-(-29x-290) \\
\hline
289
\end{array}
$$

17) $\dfrac{(2x-1)^3}{x-2}$

$\dfrac{8x^3-12x^2+6x-1}{x-2}$

$$
\begin{array}{r}
8x^2+4x+14+\dfrac{27}{x-2} \\
x-2\,\overline{)\,8x^3-12x^2+6x-1} \\
-(8x^3-16x^2) \\
\hline
4x^2+6x \\
-(4x^2-8x) \\
\hline
14x-1 \\
-(14x-28) \\
\hline
27
\end{array}
$$

18) $\dfrac{(x-3)(x^2-x-6)}{x+2}$

$\dfrac{x^3-4x^2-3x+18}{x+2}$

$$
\begin{array}{r}
x^2-6x\ +\ 9 \\
x+2\,\overline{)\,x^3-4x^2-3x+18} \\
-(x^3+2x^2) \\
\hline
-6x^2-3x \\
-(-6x^2-12x) \\
\hline
9x+18 \\
-(9x+18) \\
\hline
0
\end{array}
$$

19) $\dfrac{(x+5)^3}{x-1}$

$\dfrac{x^3+15x^2+75x+125}{x-1}$

$$
\begin{array}{r}
x^2+16x\ +\ 91+\dfrac{216}{x-1} \\
x-1\,\overline{)\,x^3+15x^2+75x+125} \\
-(x^3-\ x^2) \\
\hline
16x^2\ +75x \\
-(16x^2-16x) \\
\hline
91x+125 \\
-(91x-91) \\
\hline
216
\end{array}
$$

*Application*: Use polynomial long division to find the quotient.

20) The volume of a classroom is given by $v(x) = x^3 - 9x^2 + 23x - 15\ ft^3$. The surface area of the floor of the classroom is given by $x^2 - 4x + 3\ ft^2$. Assuming the classroom is a rectangular solid, find an expression for the height, in $ft$ of the classroom.

The area of a rectangle is given by: $A = b_{rectangle} \cdot h_{rectangle}$

The volume of a rectangular solid is given by: $V = b_{rectangle} \cdot h_{rectangle} \cdot h_{solid}$

By substitution, $V = A \cdot h_{solid}$

We divide to solve for $h_{solid}$: $\frac{V}{A} = h_{solid}$

$$
\begin{array}{r}
x - 5 \\
x^2 - 4x + 3 \overline{\left| x^3 - 9x^2 + 23x - 15 \right.} \\
\underline{-(x^3 - 4x^2 + 3x)} \\
-5x^2 + 20x - 15 \\
\underline{-(-5x^2 + 20x - 15)} \\
0
\end{array}
$$